TABLE III
ESTIMATION OF $Cp1/Cw$ AT Vds = 3 V

Expression	100% Idss	50% Idss	20% Idss
(Rs + Rd)/Rds	0.0258	0.0247	0.012
Cgd* Gm*(Rs + Rd)/ (Cgd + Cgs)	0.0278	0.0281	0.0142
$Cp1/Cw$	0.0536	0.0528	0.0252

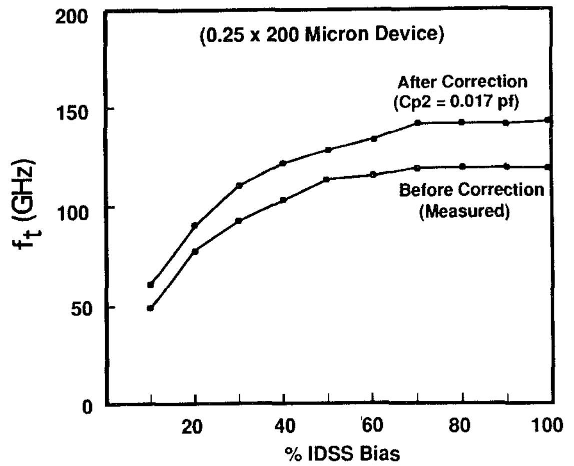

Fig. 4. f_t versus % Idss before/after parasitic capacitance correction of $Cp2/Cw$ as a function of gate biases.

in Table II at 100% Idss, 50% Idss, and 20% Idss. The (Rs + Rd)/Rds term and the Cgd*Gm*(Rd + Rs)/(Cgd + Cgs) term are calculated and listed in Table III to indicate percentage error to (Cgs + Cgd). The $Cp1/Cw = 5.4\%$ at 100% Idss and 2.5% at 20% Idss and agree well with HEMT result at high Vds as reported in [2]. $Cp2/Cw = 16\%$ and $Cp1/Cw = 5\%$ is the correction for the 0.25×200 μ gate FET and the $Cp2/Cw = 63\%$ and $Cp1/Cw = 5\%$ is the correction for the 0.25×50 μ gate FET. Consequently, $Cp2/Cw$ can be a very large correction term as gate width decreases to below 50 μ.

Next, the f_t dependence on gate bias for a 0.25×200 μ, ion implanted InGaAs MESFET is measured. We applied the above parasitic capacitance correction technique and compared the f_t before and after correction as shown in Fig. 4. Before the correction, the peak f_t is 120 GHz and the f_t values are greater than 100 GHz for biases from 40% to 100% Idss. After correction of $Cp2/Cw = 16\%$, the peak value of f_t is 143 GHz and the f_t values are greater than 100 GHz for bias ranges from 25% to 100% Idss as shown in Fig. 4. At low gate bias, the f_t correction is small because the transconductance is low. Finally, after correction of both $Cp2/Cw = 16\%$ and $Cp1/Cw = 5\%$, the peak f_t is 151 GHz for the measured f_t value of 120 GHz indicating excellent millimeter-wave performance comparable to pseudomorphic HEMT's.

VI. CONCLUSION

We have described a technique for determining the parasitic capacitance attributed to device layout geometry. This simple technique for determining the parasitic capacitance attributed to device layout geometry. This simple technique requires only on-wafer, Cascade probe measurements on devices with varying gate widths. This technique will assist in the optimization of device layout design and in improving device modeling performance for microwave and millimeter-wave applications.

REFERENCES

[1] U. K. Mishra, A. S. Brown, S. E. Rosenbaum, C. E. Hooper, M. W. Pierce, M. J. Delaney, S. Vaughn, and K. White, "Microwave performance of AlInAs-GaInAs HEMT's with 0.2 and 0.1 μm gate length," *IEEE Electron Device Lett.*, vol. 9, no. 12, p. 647, Dec. 1988.

[2] P. J. Tasker and B. Hughes, "Importance of source and drain resistance to the maximum f_t of millimeter-wave MODFET's, *IEEE Electron Device Lett.*. vol. 10, no. 8, p. 291, July 1989.

Accurate Measurement of Signals Close to the Noise Floor on a Spectrum Analyzer

Andrew A. Moulthrop and Michael S. Muha

Abstract —**Because most spectrum analyzers are calibrated to read the true power of a sinusoidal signal, a correction factor is necessary to read the true power of a nonsinusoidal signal, such as noise. Consequently, when noise and a sine wave are both present, a correction factor that is a function of the signal-to-noise ratio is necessary to find the true signal power. For some spectrum analyzers the correction factor for pure noise is incorporated into the software, but the correction factor for signal plus noise is generally ignored. This article derives this correction factor, which is significant where the signal-to-noise ratio is near unity.**

INTRODUCTION

Spectrum analyzers are commonly used to measure sinusoidal signals close to the noise floor of the measurement system. For example, measuring the third-order intercept of a small-signal amplifier requires an accurate determination of the amplitude of the third-order intermodulation products, which are usually close to the noise floor. In this case, the spectrum analyzer's internally generated distortion products, in addition to noise, can reduce the accuracy of the amplitude measurement. Since the internally generated distortion products have an unknown phase with respect to the desired signal, it is not generally possible to correct for these products; rather, one must avoid them entirely, by attenuating the input signal. Attenuating this input signal will decrease the signal-to-noise ratio, unless a feedforward cancellation technique [1] is used. In any case, noise remains as a limitation of measurement accuracy. It is possible, however, to correct for noise in the amplitude measurement, because noise causes a deterministic error. This error is deterministic because, although the noise amplitude fluctuates randomly, these fluctuations can be smoothed by using a narrow video filter or by video averaging. The signal peak can then be measured accurately; however, this power measurement includes both signal power and noise power.

One might try (naively) to correct for noise by subtracting the displayed noise power from the power of the displayed signal plus noise, but such a correction can result in a larger error than if the noise is simply ignored. This is because spectrum analyzers

Manuscript received April 11, 1991; revised July 8, 1991.
The authors are with the Aerospace Corporation, M1-110, P.O. Box 92957, Los Angeles, CA 90009.
IEEE Log Number 9103095.

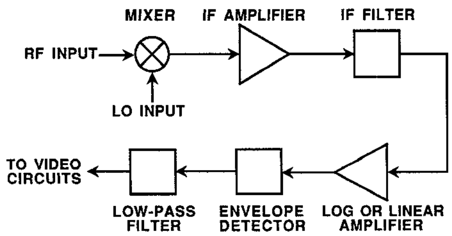

Fig. 1. Block diagram of the IF-detector portion of a typical spectrum analyzer.

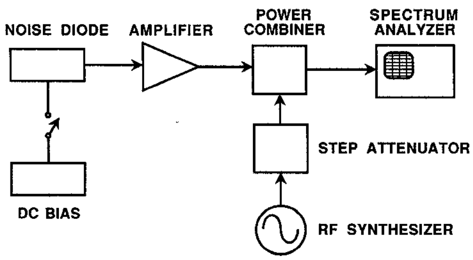

Fig. 2. Block diagram of a signal-plus-noise measurement system.

are normally calibrated to display the true power level of only *sinusoidal* signals, not that of Gaussian noise. However, to find the true signal amplitude when noise and a sinusoidal signal are both present, a correction factor must be applied that is a complicated function of the displayed signal-to-noise ratio. This correction, derived below, depends on the displayed signal-to-noise ratio, as well as on whether the spectrum analyzer's receiver uses a logarithmic or a linear amplifier.

Analysis and Experiment

A simplified block diagram [2] for the IF-detector portion of a typical spectrum analyzer is shown in Fig. 1. Either the log or linear amplifier is selected, depending on whether the spectrum analyzer is in the logarithmic or linear display mode, respectively. The amplitudes displayed on the spectrum analyzer are proportional to the output voltages of the low-pass filter. We are interested in the output voltage for a signal plus noise compared to that for noise alone and for the signal alone. To analyze this measurement, we construct a conceptual model that includes both a sinusoidal signal and a Gaussian noise source that is band-limited by the IF (resolution) filter. The sinusoidal signal has an amplitude A just before the envelope detector. The total noise power at the same point has the value N, which includes any noise generated in the spectrum analyzer. An experimental realization of this model is depicted schematically in Fig. 2. Below we derive the corrections for the linear and the logarithmic modes separately.

Linear Mode

For the Gaussian noise plus the signal of our model, it has been shown [3] that the probability density of the envelope is

$$p(E)=\frac{E}{N}\exp\left(-\frac{1}{2N}(A^2+E^2)\right)I_0\left(\frac{EA}{N}\right) \tag{1}$$

where E is the envelope voltage and $I_0(EA/N)$ is the modified Bessel function of the first kind and zeroth order. The output voltage of the low-pass filter following the envelope detector, if the cutoff frequency is low enough to remove all but the dc component, is the average value of the output voltage of the envelope detector:

$$\begin{aligned}\overline{E}_{\text{lin}}(N,m)&=\int_0^\infty Ep(E)\,dE\\&=\frac{\sqrt{N}}{(2m)^{3/2}}\exp(-m)\int_0^\infty z^2\exp\left(-\frac{z^2}{4m}\right)I_0(z)\,dz.\end{aligned} \tag{2}$$

We have changed the variables on the right-hand side of (2), using

$$m=\frac{A^2}{2N}\qquad z=\sqrt{\frac{2m}{N}}\,E \tag{3}$$

to make the average envelope voltage $\overline{E}_{\text{lin}}$ a function of the noise power N and the signal-to-noise ratio m. The integration can be performed by writing $I_0(z)$ as an ascending series, so that

$$I_0(z)=\sum_{k=0}^{\infty}\frac{\left(\frac{1}{4}z^2\right)^k}{(k!)^2}. \tag{4}$$

Putting (4) into (2) and carrying out the integration, we get

$$\overline{E}_{\text{lin}}(N,m)=\sqrt{\frac{N\pi}{2}}\exp(-m)\cdot\sum_{k=0}^{\infty}\left(\frac{m}{2}\right)^k\frac{[1\cdot3\cdot5\cdot\cdots\cdot(2k+1)]}{(k!)^2}. \tag{5}$$

If there is no signal, but rather only noise of average power N, then (5) becomes

$$\overline{E}_{\text{lin}}(N,0)=\sqrt{\frac{N\pi}{2}}. \tag{6}$$

On the other hand, if there is no noise, but rather only a sinusoidal signal of average power N, (5) becomes

$$\lim_{m\to\infty}\overline{E}_{\text{lin}}\left(\frac{N}{m},m\right)=\sqrt{2N}. \tag{7}$$

Spectrum analyzers are calibrated to display the correct voltage for a sinusoidal signal. However, by dividing (6) by (7), we see that the ratio of the envelope voltage of noise to the envelope voltage of a sinusoidal signal having the same average power is

$$\sqrt{\frac{\pi}{4}}=-1.05\text{ dB}. \tag{8}$$

Hence, as a result of envelope detection, the voltage of noise alone as displayed on a spectrum analyzer operating in the linear mode will be 1.05 dB lower than the true noise voltage. The noise power per resolution bandwidth as displayed on a spectrum analyzer must also be corrected (in both the linear and logarithmic modes) to convert the power-transfer curve of the resolution filter to that of an ideal rectangular filter. Both of these corrections are discussed in greater detail elsewhere [2].

In contrast, the correction to be applied when both signal and noise are present is described only briefly in [2]. Two ratios are useful in discussing this correction. One is the ratio between 1) the voltage displayed at the signal frequency in the presence of both signal and noise to 2) the noise voltage displayed on the spectrum analyzer with no signal applied ($m=0$):

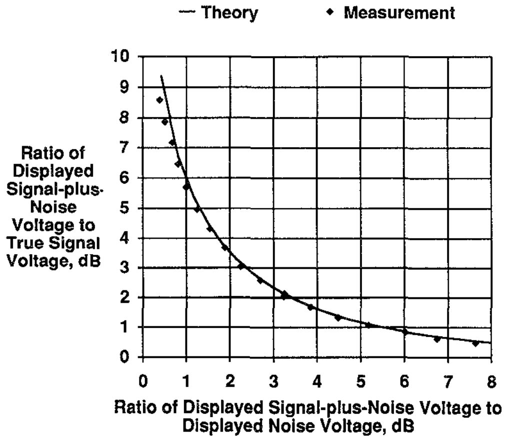

Fig. 3. Effect of noise on the displayed signal voltage when a spectrum analyzer is in linear display mode.

$$R_{snn} \equiv \frac{\text{(displayed signal plus noise voltage)}}{\text{(displayed noise voltage)}} = \exp(-m) \sum_{k=0}^{\infty} \left(\frac{m}{2}\right)^k \frac{[1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2k+1)]}{(k!)^2}. \quad (9)$$

The other useful ratio is that of 1) the displayed signal-plus-noise voltage to 2) the displayed signal voltage with noise absent:

$$R_{sns} \equiv \frac{\text{(displayed signal plus noise voltage)}}{\text{(displayed signal voltage if noise is absent)}} = \sqrt{\frac{\pi}{4m}} \exp(-m) \sum_{k=0}^{\infty} \left(\frac{m}{2}\right)^k \frac{[1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2k+1)]}{(k!)^2}. \quad (10)$$

The displayed signal voltage with noise absent is also the true signal voltage. Hence R_{sns} is the ratio of 1) the displayed voltage at the signal frequency with noise present to 2) the true signal voltage one would measure in absence of noise. Experimentally, one can measure R_{snn}, solve (9) for m, and then use that value for m to find R_{sns}. The displayed signal-plus-noise voltage is then divided by R_{sns} to give the true signal voltage. This procedure is simplified by plotting R_{sns} versus R_{snn}; this is done in Fig. 3, where a logarithmic scale is used. Fig. 3 shows both the theoretical curve, obtained by plotting R_{sns} versus R_{snn}, and measured data, obtained by using a Hewlett-Packard 8566B spectrum analyzer in the experimental setup depicted in Fig. 2. To use Fig. 3, first find the ratio of displayed signal plus noise voltage to displayed noise voltage on the horizontal axis; the corresponding value on the vertical axis is the correction factor, which, when subtracted from the displayed signal-plus-noise voltage, yields the true signal voltage.

For example, say one measures a noise voltage of 100 μV and a signal-plus-noise voltage of 141 μV for a ratio of signal plus noise to noise of 3 dB. Fig. 3 shows that the corresponding ratio of displayed signal-plus-noise voltage to true signal voltage is approximately 2.3 dB; hence, the true signal voltage is 2.3 dB below the measured signal-plus-noise voltage, or 108 μV. Note that this correction depends directly on the displayed noise voltage, which itself is a function of the the true noise voltage, the resolution bandwidth, and the shape of the resolution filter. Therefore, to find the true signal voltage in the presence of noise, one need not be concerned with the true noise voltage or the characteristics of the resolution filter, but only with the displayed value of noise. Similarly, for the logarithmic mode (discussed below), the correction depends directly on the displayed noise power, so one need not be concerned with the true noise power or the characteristics of the resolution filter.

Logarithmic Mode

Note that the log amplifier shown in Fig.1 can not perform a mathematical log operation on the input voltage, since the input voltage can be negative. Rather, this amplifier is linear except that gain is controlled such that the output amplitude is proportional to the log of the input amplitude. For example, an input sinusoidal signal of amplitude A will become a sinusoidal signal of amplitude $G(lnA)$, where G is the gain constant. Hence the envelope of the output of the log amplifier is equal to $G(lnE)$, where E is the envelope of the input signal. Equation (2) is therefore modified for this case because instead of averaging the envelope of the input signal, we must average the log of the envelope of the input signal; thus,

$$\bar{E}_{\log}(N,m) = G\int_0^{\infty} (lnE)\, p(E)\, dE = \frac{G}{2m} \exp(-m) \cdot \int_0^{\infty} \left(lnz + \frac{1}{2} ln \frac{N}{2m}\right) z \exp\left(-\frac{z^2}{4m}\right) I_0(z)\, dz \quad (11)$$

Note that the same result would be obtained if in Fig. 1 the log amplifier preceeding the envelope detector were removed and replaced with an amplifier that performs the mathematical log operation following the envelope detector. Since we are using the natural logarithm in (11), the constant G must be 8.68 to give a display in decibels. The integration in (11) can be performed by using the ascending series of (4) to arrive at

$$\bar{E}_{\log}(N,m) = \frac{G}{2}\left(ln 2N - \gamma + \exp(-m) \cdot \sum_{k=1}^{\infty} \frac{m^k}{k!}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right)\right) \quad (12)$$

where $\gamma = 0.577216\ldots$ is Euler's constant.

Using (12) for the log mode, we can find the ratio that corresponds to (8) for the linear mode. That is, the ratio of the displayed noise power to the displayed signal power for equal average power is $2/G\gamma = -2.507$ dB. Note that (12) is in decibels, so it is convenient to find the ratio in dB. Since a spectrum analyzer is calibrated to read the correct power for a sinusoidal signal, log amplification and envelope detection thus cause the average power of noise alone (as displayed on a spectrum analyzer operating in the logarithmic mode) to be 2.507 dB lower than the true noise power. This correction is described in greater detail elsewhere [2].

Also using (12) for the log mode, we can find the ratios that correspond to R_{snn} and R_{sns} for the linear mode. In the log mode the ratio corresponding to R_{snn} is

$$\rho_{snn} \equiv \frac{\text{(displayed signal plus noise voltage)}}{\text{(displayed noise voltage)}} = \frac{G}{2} \exp(-m) \sum_{k=1}^{\infty} \frac{m^k}{k!}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right) \quad (13)$$

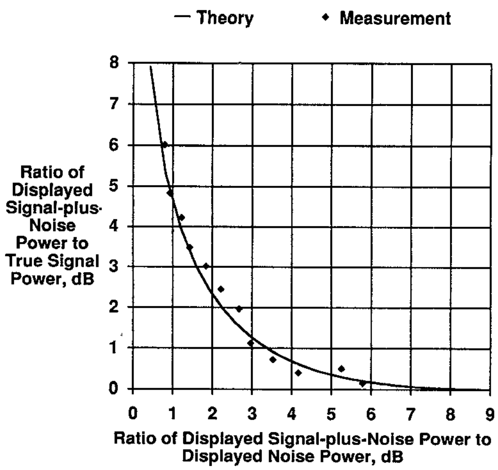

Fig. 4. Effect of noise on the displayed signal power when a spectrum analyzer is in logarithmic display mode.

Similarly, the ratio for the log mode corresponding to R_{sns} is

$$\rho_{\text{sns}} \equiv \frac{(\text{displayed signal plus noise voltage})}{(\text{displayed signal voltage if noise is absent})}$$

$$= \frac{G}{2}\left(- \ln m - \gamma + \exp(-m) \cdot \sum_{k=1}^{\infty} \frac{m^k}{k!}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right)\right). \tag{14}$$

A plot of ρ_{sns} versus ρ_{snn} is given in Fig. 4, along with measured data obtained on a Hewlett-Packard 8566B spectrum analyzer in the setup depicted in Fig. 2. As we did with Fig. 3 for the linear mode, we can use Fig. 4 for the logarithmic mode, as follows. First, find the measured ratio of displayed signal plus noise to displayed noise on the horizontal axis; the corresponding value on the vertical axis is the correction factor, which, when subtracted from the displayed signal-plus-noise power, yields the true signal power. For example, say one measures a noise power of -63 dBm and a signal-plus-noise power of -60 dBm for a ratio of signal plus noise to noise of 3 dB. Fig. 4 shows that the corresponding ratio of displayed signal-plus-noise power to true signal power is approximately 1.25 dB; hence the true signal voltage is 1.25 dB below the measured signal-plus-noise voltage, or -61.25 dBm.

Conclusion

A spectrum analyzer does not measure true power, but rather only the envelope of the voltage in the linear mode, or the log of the envelope of the voltage in the log mode. The corrections to be applied to arrive at the true signal power when noise is present are derived above. The corrections are shown graphically in Fig. 3 for the linear mode, and in Fig. 4 for the log mode. To use these graphs one finds the ratio of displayed signal plus noise to displayed noise on the horizontal scale; then the corresponding value on the vertical scale is the number of dB one must subtract from the displayed signal plus noise to arrive at the true signal level.

Acknowledgment

The authors wish to acknowledge W. A. Johnson, A. M. Young, and M. T. Meyer for their valuable comments, and C. P. Silva for suggesting analytic solutions to the integrals used in the above analysis.

References

[1] D. F. Snyder, "Improving the resolution of IM and other distortion measurements," *Microwave Journal*, pp. 352–353, May 1986.
[2] B. Peterson, "Spectrum analyzer basics,"Hewlett-Packard Application Note 150, 1989.
[3] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw-Hill, 1966, p. 26.

Synthesis of Schiffman Phase Shifters

José Luis Ramos Quirarte and J. Piotr Starski

***Abstract*—The Schiffman phase shifter is a very useful passive component. In this paper equations to determine its phase deviation and maximum bandwidth when the coupling coefficient is known are presented. Equations are given also to determine the coupling coefficient for a desired bandwidth or maximum phase deviation.**

***Keywords*—Phase shifters; Schiffman sections; coupled transmission lines.**

I. Introduction

The Schiffman phase shifter [1]–[5] is a broadband differential phase shifter where the phase shift $\Delta\phi$, is obtained as a subtraction of the phase response of a coupled section with adjacent ports interconnected [6] and the phase response of a uniform line. i.e.:

$$\Delta\phi = K\theta - \cos^{-1}\left(\frac{\rho - \tan^2\theta}{\rho + \tan^2\theta}\right) \tag{1}$$

where θ is the electrical length of the coupled section and ρ is its *impedance ratio* defined as

$$\rho = \frac{Z_{0e}}{Z_{0o}}. \tag{2}$$

Z_{0e} and Z_{0o} are the even and odd mode impedances of the coupled section respectively. The input impedance Z_I, of the coupled section is matched at all frequencies and is given by

$$Z_I = \sqrt{Z_{0e} Z_{0o}}. \tag{3}$$

The *coupling* C, and the impedance ratio are related by [2], [7]:

$$C = -20 \log \frac{\rho - 1}{\rho + 1} \tag{4}$$

or

$$\rho = \frac{1 + 10^{-C/20}}{1 - 10^{-C/20}}. \tag{5}$$

Manuscript received February 19, 1991; revised May 30, 1991.
The authors are with the Division of Network Theory, Chalmers University of Technology, 412 96, Gothenburg, Sweden.
IEEE Log Number 9102821.